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PROPAGATION OF LINEAR WAVES IN GAS-SATURATED POROUS MEDIA WITH ALLOWANCE FOR INTERPHASE HEAT TRANSFER

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The effect of gas–skeleton heat-transfer processes on propagation of fast and slow waves in a porous medium is examined. Frequency intervals are identified, in which attenuation of waves in a gas-saturated porous medium is mainly controlled by the heat-transfer processes.

Key words: *porous medium, linear waves, dispersion relation, wavenumber, heat transfer.*

Introduction. The majority of natural and technological media are not homogeneous and, hence, cannot be classified as liquids, gases, or deformable solids. The differences in the properties of individual phases composing a medium and the interphase interactions play the governing role in the dynamics of such media.

Acoustic methods offer most powerful tools for studying such formations. In particular, based on an analysis of the echo signal related to the structure and properties of the medium under study, one can monitor many processes in porous media (restoration of natural permeability, prevention and resolving of problems related to liquid adsorption, etc.).

Theoretical and experimental studies of acoustic-wave propagation in porous media present an urgent matter of significance for gaining a better insight into processes that accompany the use of advanced technologies dealing with porous media. There are many reported studies concerning the acoustics of porous media and propagation of waves in such media [1–10].

A study of weak disturbances propagating in a deformable double-porosity medium saturated with a liquid was reported by Gubaidullin and Kuchurugina [1]. In such media, one transverse and three longitudinal waves, namely, one deformation wave and two filtration waves, were found to propagate. The velocity of the waves was considered as a function of interphase force interaction.

Egorov et al. [2, 3] considered the propagation of monochromatic waves in thin-layered saturated media by averaging differential equations with rapidly oscillating coefficients. Primary attention was paid to the transformation attenuation mechanism for such waves. Smirnov and Safargulova [4] examined the structure of waves in porous media. Gubaidullin et al. [5] considered the penetration of a stepwise wave through the gas–porous medium interface and its reflection from a rigid wall covered by a porous material. The effect of porous-medium and wave parameters on the reflection process was examined.

In the works cited above, the thermal effects due to the interphase interaction, which may appear significant, were ignored.

The influence of interphase heat- and mass-transfer processes on propagation of weak disturbances in foams was considered by Shagapov [6], who obtained a dispersion relation for waves and examined their phase velocity and attenuation factors as functions of medium and disturbance parameters.

In the present work, we examine the influence of interphase thermal effects and interphase forces on propagation of waves in porous media. The thermal effects are taken into account for the first time.

Basic Equations. We consider a porous medium (e.g., a sponge) saturated with a gas. In analyzing the propagation of one-dimensional waves in such media, we assume that the wave length is much greater than the size of pores in the medium.

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We write the linearized macroscopic continuity equations for the skeleton of the porous medium and for the gas in the pores in the two-velocity approximation:

$$\frac{\partial \rho_j}{\partial t} + \rho_{j0}\,\frac{\partial v_j}{\partial x} = 0. \tag{1}$$

Here, ρ_j and v_j are, respectively, the density and velocity of the jth phase. The subscript $j = \mathrm{s}, \mathrm{g}$ refers to skeleton parameters and to the parameters of the gas in the pores, and the additional subscript 0 refers to the initial state.

The momentum equation for the system as a whole is

$$\rho_{\mathrm{g},0}\,\frac{\partial v_{\mathrm{g}}}{\partial t} + \rho_{\mathrm{s},0}\,\frac{\partial v_{\mathrm{s}}}{\partial t} = \alpha_{\mathrm{s},0}\,\frac{\partial \sigma_{\mathrm{s}}}{\partial x} - \alpha_{\mathrm{g},0}\,\frac{\partial p_{\mathrm{g}}}{\partial x}, \tag{2}$$

where p_{g} is the gas pressure, α_{s} and α_{g} are the volume contents of the solid and gas phases, respectively, and σ_{s} is the stress. For the skeleton, we adopt the Voigt model [11]. Then, we have

$$\sigma_{\mathrm{s}} = E_{\mathrm{s}}\varepsilon + \mu_{\mathrm{s}}\,\frac{\partial \varepsilon}{\partial t}, \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v_{\mathrm{s}}}{\partial x}, \tag{3}$$

where E_{s} and μ_{s} are the elastic modulus of the porous skeleton and its dynamic viscosity, respectively. The momentum equation for the gas phase is [7]

$$\rho_{\mathrm{g},0}\,\frac{\partial v_{\mathrm{g}}}{\partial t} = -\alpha_{\mathrm{g},0}\,\frac{\partial p_{\mathrm{g}}}{\partial x} - F, \qquad F = F_m + F_\mu + F_B, \tag{4}$$

where

$$F_m = \frac{1}{2}\,\eta_m \alpha_{\mathrm{g},0}\alpha_{\mathrm{s},0}\rho_{\mathrm{g}}^0\Big(\frac{\partial v_{\mathrm{g}}}{\partial t} - \frac{\partial v_{\mathrm{s}}}{\partial t}\Big), \qquad F_\mu = \frac{9}{2}\,\eta_\mu \alpha_{\mathrm{g},0}\alpha_{\mathrm{s},0}\mu_{\mathrm{g}}(v_{\mathrm{g}} - v_{\mathrm{s}})a_0^{-2},$$

$$F_B = 6\eta_B a_0^2\sqrt{\pi\rho_{\mathrm{g}}^0\mu_{\mathrm{g}}}\int\limits_{-\infty}^{t} \frac{\partial}{\partial \tau}\,(v_{\mathrm{g}} - v_{\mathrm{s}})\,\frac{d\tau}{\sqrt{t-\tau}}.$$

Here, F_m is the virtual-mass force due to inertial interaction between the phases, F_μ is the analog of the Stokes viscous-friction force, F_B is the Basset force due to nonstationary effects, μ_{g} is the dynamic viscosity of the gas, and η_m, η_μ, and η_B are some coefficients that depend on the parameters of the porous medium [1].

The heat-dissipation processes in the system under study are determined by the temperature distribution in the vicinity of the interfaces between the phases. To describe the temperature-field micrononuniformities, we use the medium-structure schematization proposed in [7]. We consider the gas-saturated porous medium as a system of spherical gas bubbles surrounded by a skeleton-material layer. Thus, at each macroscopic point defined by the coordinate x, we introduce a typical cell that consists of a gas bubble and the adjacent part of the skeleton. Inside the cell, we have some temperature distribution $T_j'(t,x,r)$ and some gas-density distribution $\rho_{\mathrm{g}}'^0(t,x,r)$, where r is the coordinate reckoned from the center of the cell.

The relation between the density $\rho_{\mathrm{g}}'^0(t,x,r)$ and the true mean density $\rho_{\mathrm{g}}^0(t,x)$ for the gas phase is given by the expression

$$\rho_{\mathrm{g}}^0 = \frac{3}{4\pi a^3}\int\limits_0^a \rho_{\mathrm{g}}'^0 4\pi r^2\,dr,$$

where a is the pore radius.

In addition, for the true densities ρ_j^0 and for the volume phase contents α_j, we can write the kinematic relations

$$\rho_j = \alpha_j\rho_j^0, \qquad \alpha_{\mathrm{g}} = a^3/(a+b)^3, \qquad \alpha_{\mathrm{g}} + \alpha_{\mathrm{s}} = 1,$$

where b is the half-thickness of the pore wall.

If the condition $b \ll a$ ($\alpha_{\mathrm{s}} \ll 1$) holds, we have $\alpha_{\mathrm{s}} = 3b/a$.

To describe the temperature distribution in a cell of the porous medium, we write the following linearized heat-conduction equations [6]:

$$\rho_{\mathrm{g},0}^0 c_{\mathrm{g}}\,\frac{\partial T_{\mathrm{g}}'}{\partial t} = \lambda_{\mathrm{g}} r^{-2}\,\frac{\partial}{\partial r}\Big(r^2\,\frac{\partial T_{\mathrm{g}}'}{\partial r}\Big) + \frac{\partial p_{\mathrm{g}}}{\partial t} \qquad (0 < r < a_0); \tag{5}$$

$$\rho_{\mathrm{s},0}^0\, c_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}'}{\partial t} = \lambda_{\mathrm{s}}\,\frac{\partial^2 T_{\mathrm{s}}'}{\partial r^2} \qquad (a_0 < r < a_0 + b_0). \tag{6}$$

Here λ_j and c_j are, respectively, the thermal conductivity and the specific heat capacity at constant pressure ($j = \mathrm{g}, \mathrm{s}$), a_0 is the mean pore radius, and b_0 is the mean half-thickness of the pore walls.

With regard for the temperature and heat-flux continuity at the interface between the phases $r = a_0$, we write the boundary conditions at this interface for Eqs. (5) and (6) as

$$T'_{\mathrm{g}} = T'_{\mathrm{s}}, \qquad \lambda_{\mathrm{s}} \frac{\partial T'_{\mathrm{s}}}{\partial r} = \lambda_{\mathrm{g}} \frac{\partial T'_{\mathrm{g}}}{\partial r} \qquad (r = a_0). \tag{7}$$

We write the condition of temperature finiteness at the centers of the bubbles and the condition of no heat transfer between the cells (condition of cell adiabaticity):

$$\frac{\partial T'_{\mathrm{g}}}{\partial r} = 0 \quad (r = 0), \qquad \frac{\partial T'_{\mathrm{s}}}{\partial r} = 0 \quad (r = a_0 + b_0). \tag{8}$$

We assume that the gas contained in the pores of the medium is calorifically perfect. Then,

$$p_{\mathrm{g}} = \rho'^{0}_{\mathrm{g}} R T'_{\mathrm{g}}, \tag{9}$$

where R is the universal gas constant.

Propagation of Linear Waves in the Porous Medium. We seek the solution of system (1)–(6), (9) in the form of decaying running waves

$$\rho^0_j, v_j, p_j, \alpha_j \cong \exp{[i(Kx - \omega t)]}, \qquad T'_j = A_{T_j}(r) \exp{[i(Kx - \omega t)]}, \qquad K = k + i\delta, \tag{10}$$

where ω is the circular frequency, K is the complex wavenumber, $C_p = \omega/k$ is the mean phase velocity, and δ is the attenuation factor.

We obtain the expressions for the distributions of temperatures T'_{g} and T'_{s} by solving system (5), (6) with allowance for conditions (7) and (8):

$$A_{T'_{\mathrm{g}}} = T_0(1 - \gamma^{-1}) \Big[1 - A \frac{\sinh{(y_{\mathrm{g}} r/a_0)}}{\sinh{(y_{\mathrm{g}})}} \frac{a_0}{r}\Big] \frac{A_{p_{\mathrm{g}}}}{P_0},$$

$$A_{T'_{\mathrm{s}}} = T_0(1 - \gamma^{-1}) \frac{\cosh{[y_{\mathrm{s}}(a_0 + b_0 - r)/b_0]}}{\cosh{(y_{\mathrm{s}})}} \frac{A_{p_{\mathrm{g}}}}{P_0} (1 - A),$$

$$A = 1/[1 + y_{\mathrm{s}} \coth{(y_{\mathrm{s}})} \Pi_{\mathrm{g}}(y_{\mathrm{g}})/\eta], \qquad \Pi_{\mathrm{g}}(y_{\mathrm{g}}) = 3[y_{\mathrm{g}} \coth{(y_{\mathrm{g}})} - 1] y_{\mathrm{g}}^{-2},$$

$$y_{\mathrm{g}} = \sqrt{-\frac{i\omega a_0^2}{æ_{\mathrm{g}}}}, \qquad y_{\mathrm{s}} = \sqrt{-\frac{i\omega b_0^2}{æ_{\mathrm{s}}}}, \qquad æ_{\mathrm{g}} = \frac{\lambda_{\mathrm{g}}}{\rho^0_{\mathrm{g},0} c_{\mathrm{g}}}, \qquad æ_{\mathrm{s}} = \frac{\lambda_{\mathrm{s}}}{\rho^0_{\mathrm{s},0} c_{\mathrm{s}}}, \qquad \eta = \frac{\alpha_{\mathrm{s},0} \rho^0_{\mathrm{s},0} c_{\mathrm{s}}}{\alpha_{\mathrm{g},0} \rho^0_{\mathrm{g},0} c_{\mathrm{g}}}.$$

From the condition of existence of a solution of the form (10), after some re-arrangements, we obtain the following dispersion relation:

$$\frac{K}{\omega} = \pm \frac{1}{C_{\mathrm{g}} \sqrt{2}} \sqrt{B_1 + B_2 \tilde{C}^2 \pm \sqrt{(B_1 + B_2 \tilde{C}^2)^2 - 4 B_3 \tilde{C}^2}}. \tag{11}$$

Here

$$B_1 = (1 + \chi_T)(1 + i\chi_V \alpha_{\mathrm{s},0}), \qquad B_2 = \frac{1 + i\beta\chi_V}{1 - i\omega\mu_{\mathrm{s}}/E_{\mathrm{s}}}, \qquad B_3 = \frac{(1 + \chi_T)(i\chi_V(\alpha_{\mathrm{s},0} + \beta\alpha_{\mathrm{g},0}) + 1)}{1 - i\omega\mu_{\mathrm{s}}/E_{\mathrm{s}}},$$

$$\beta = \frac{\rho^0_{\mathrm{g},0}}{\rho^0_{\mathrm{s},0}}, \quad \tilde{C} = \frac{C_{\mathrm{g}}}{C_{\mathrm{s}}}, \quad C_{\mathrm{g}} = \sqrt{\frac{\gamma P_0}{\rho^0_{\mathrm{g},0}}}, \quad C_{\mathrm{s}} = \sqrt{\frac{E_{\mathrm{s}}}{\rho^0_{\mathrm{s},0}}}, \quad \chi_T = (\gamma - 1) A\, \Pi_{\mathrm{g}}(y_{\mathrm{g}}), \quad \chi_V = \frac{1}{\omega\tau^*},$$

$$\tau^{*-1} = -i\omega\eta_m \alpha_{\mathrm{g},0}\alpha_{\mathrm{s},0}/2 + \eta_\mu \alpha_{\mathrm{g},0}\alpha_{\mathrm{s},0} \nu_{\mathrm{g}} a_0^{-2} + \eta_B (1 + i) \alpha_{\mathrm{g},0}\alpha_{\mathrm{s},0} a_0^{-1} \sqrt{2\nu_{\mathrm{g}}\omega}, \qquad \nu_{\mathrm{g}} = \mu_{\mathrm{g}}/\rho^0_{\mathrm{g},0},$$

where C_{g} and C_{s} are the phase velocities of the wave in the gas and in the skeleton, and ρ^0_{j0} is the initial true density of the jth phase.

The coefficients χ_V and χ_T allow for the effect of nonstationary interphase interaction forces and the effect of heat exchange between the gas and the skeleton on the dynamics of the fast and slow waves.

Calculation Results. From the dispersion relation (11), we calculated the mean phase velocities and the attenuation factors of both waves. The calculations were carried out for the rubber–air (porous medium of the sponge type) and rubber–hydrogen systems. The parameters of the phases, taken for an ambient temperature of 300 K, were as follows: $\rho^0_{\mathrm{s},0} = 920$ kg/m^3, $E_{\mathrm{s}} = 10^8$ Pa, $\lambda_{\mathrm{s}} = 0.15$ W/(m $\cdot$ K), $c_{\mathrm{s}} = 1571$ J/(kg $\cdot$ K), and $\mu_{\mathrm{s}} = 100$ Pa $\cdot$ sec

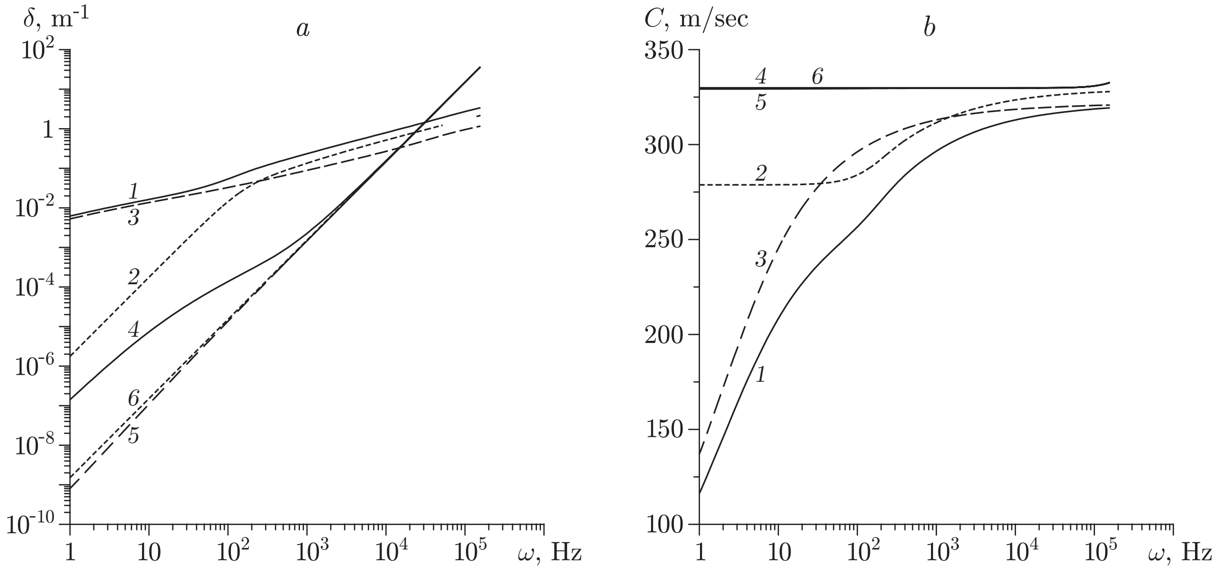

Fig. 1. Attenuation factor (a) and phase velocity (b) of the slow and fast waves versus frequency for $a_0 = 10^{-3}$ m and $b_0 = 3.57 \cdot 10^{-5}$ m.

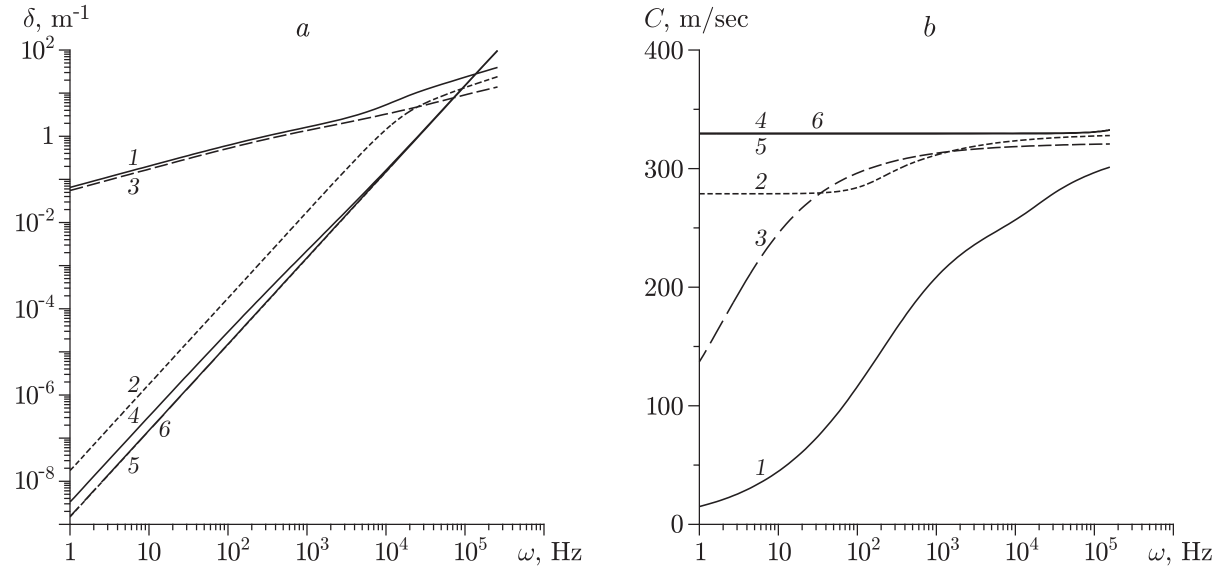

Fig. 2. Attenuation factor (a) and phase velocity (b) of the slow and fast waves versus frequency for $a_0 = 10^{-4}$ m and $b_0 = 3.57 \cdot 10^{-6}$ m.

for rubber; $\rho^0_{\mathrm{g},0} = 1.29$ kg/m^3, $c_\mathrm{g} = 1006$ J/(kg $\cdot$ K), $\lambda_\mathrm{g} = 0.025$ W/(m $\cdot$ K), $\gamma = 1.4$, and $\mu_\mathrm{g} = 1.86 \cdot 10^{-5}$ Pa $\cdot$ sec for air; $\rho^0_{\mathrm{g},0} = 0.09$ kg/m^3, $\lambda_\mathrm{g} = 0.17$ W/(m $\cdot$ K), $\mu_\mathrm{g} = 0.84 \cdot 10^{-5}$ Pa $\cdot$ sec, $c_\mathrm{g} = 14284$ J/(kg $\cdot$ K), and $\gamma = 1.41$ for hydrogen. In the calculations, satisfaction of the continuity equation was verified; this condition requires the wave length to be greater than the characteristic size of medium nonuniformity.

Figures 1 and 2 show the phase velocity and the attenuation factor of the slow (curves 1, 2, and 3) and fast (curves 4, 5, and 6) waves versus frequency for the rubber–air system. Curves 2 and 6 were calculated with allowance for heat transfer and disregarding the interphase forces, curves 3 and 5 were obtained with allowance for the interphase forces and disregarding heat transfer, and curves 1 and 4 were calculated with allowance for both heat transfer and the interphase forces. Here and below, the gas content was assumed to be $\alpha_{\mathrm{g},0} = 0.9$.

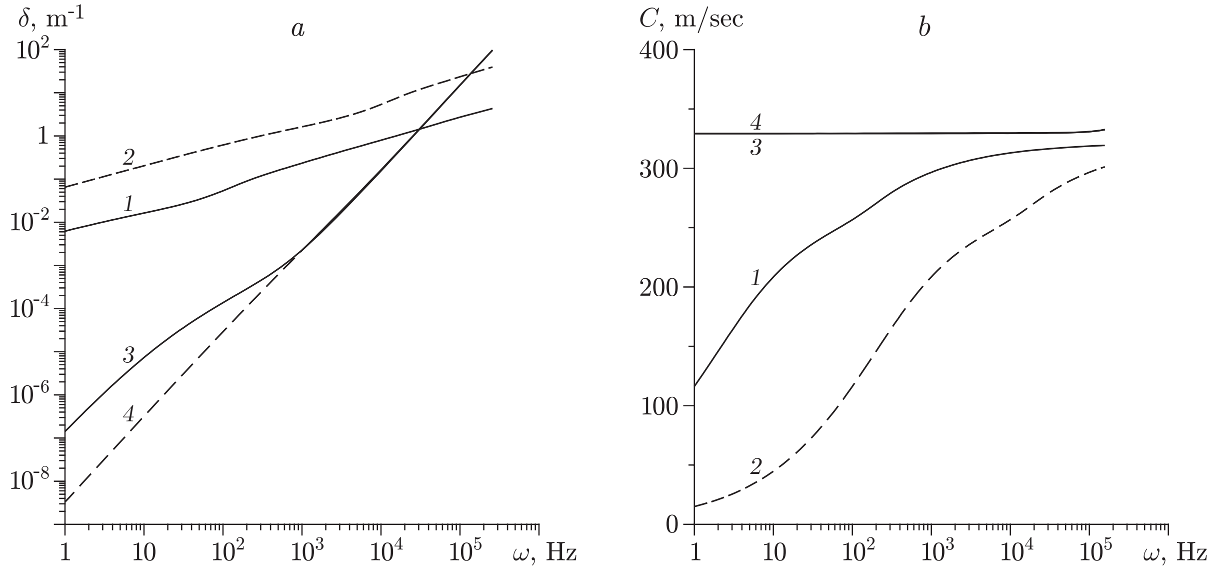

Fig. 3. Effect of the pore size on the attenuation factor (a) and phase velocity (b) of the slow and fast waves.

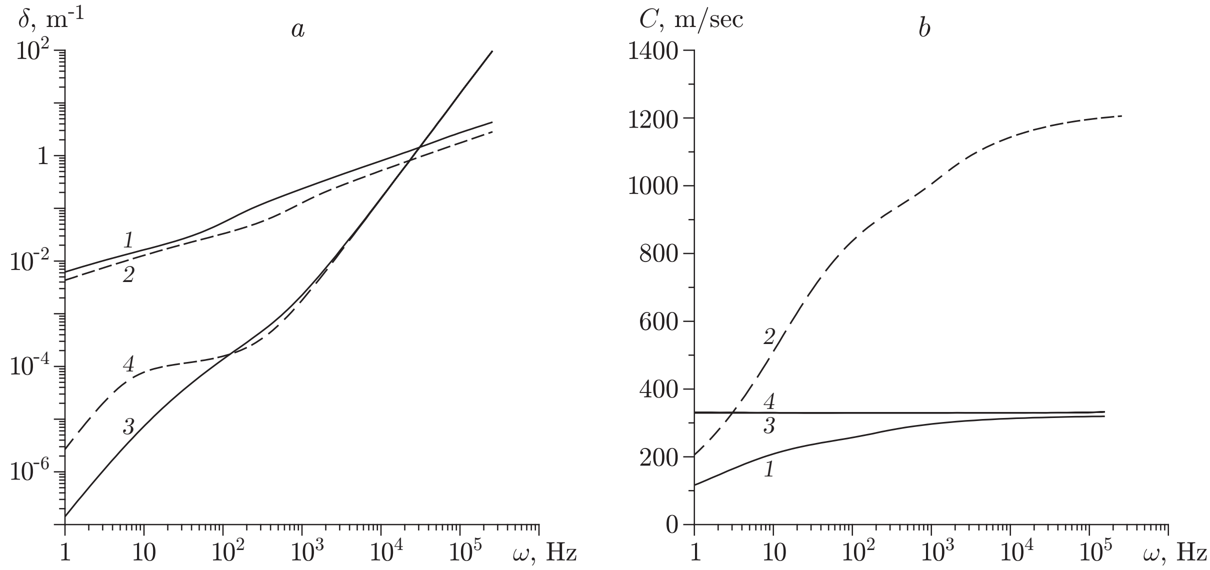

Fig. 4. Attenuation factor (a) and phase velocity (b) of the slow and fast waves versus frequency for the rubber–air and rubber–hydrogen systems for $a_0 = 10^{-3}$ m.

It follows from Figs. 1 and 2 that the heat-transfer process strongly affects attenuation of fast waves in the low-frequency region; the effect of heat transfer on attenuation of slow waves becomes noticeable at frequencies above $\omega^* \approx 10^2$ sec^{-1}. A tenfold decrease in the characteristic pore size results in a 20-fold increase in the characteristic frequency ω^*. At higher frequencies ($\omega \approx 10^6$ sec^{-1}), the effect of heat transfer and interphase forces on the velocity of the slow wave is insignificant.

Figure 3 illustrates the effect of pore sizes on the velocity and attenuation factor of the slow wave (curves 1 and 2) and fast wave (curves 3 and 4). Curves 1, 3 and 2, 4 refer to the pore sizes $a_0 = 10^{-3}$ m and $a_0 = 10^{-4}$ m, respectively. As the pore size decreases, the velocity of the slow wave in the gas decreases, whereas the velocity of the fast wave weakly depends on the pore size.

As the pore size decreases by a factor of ten, the attenuation rate of the slow wave displays a tenfold decrease. The fast wave in a medium with large pores decays more rapidly in the low-frequency region; above the frequency of $\omega^* \sim 10^3$ sec^{-1}, the attenuation rate of this wave for the pore sizes considered remains almost unchanged.

Figure 4 shows the velocity and attenuation factor of the slow wave (curves 1 and 2) and fast wave (curves 3 and 4) versus frequency. Curves 1 and 3 refer to the rubber–air porous medium, and curves 2 and 4 refer to the rubber–hydrogen system. The slow wave in the rubber–air system is seen to decay more rapidly than in the rubber–hydrogen system. The fast wave decays more rapidly in the rubber–hydrogen porous medium in the low-frequency region; in the frequency range $\omega^* \geqslant 10^2$ sec^{-1}, the attenuation in both cases is almost identical. The velocity of the slow wave in hydrogen is greater than in air.

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